Calculus Warm Up # 6-2

Simplify:

1) $\sqrt{1+2x+x^2}$

2) $\sqrt{1-x+\frac{x^2}{4}}$

3) $\sqrt{1+\left(\frac{x}{4}-\frac{1}{x}\right)^2}$

4) $\sqrt{1+\cos 2x}$

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In Exercises 1 and 2, find the distance between the given points by (a) using the distance formula and (b) determining the equation of the line through the points and using the formula for arc length.

1. (0, 0), (5, 12)

b) $y = \frac{12}{5}x$

$y' = \frac{12}{5}$

In Exercises 3–8, find the arc length of the graph of the given function over the indicated interval.

3. $y = \frac{2}{3}x^{3/2} + 1 \qquad [0, 1]$

5. $y = \frac{x^4}{8} + \frac{1}{4x^2}$ $[1, 2]$

$y' = \frac{1}{2}(x^3 - x^{-3})$

$(y')^2 = \frac{1}{4}(x^6 - 2 + \frac{1}{x^6})$

$S = \int$

7. $y = \frac{x^5}{10} + \frac{1}{6x^3}$ $[1, 2]$

$(y' = \frac{x^4}{2} - \frac{x^{-4}}{2})^2$

$(y')^2 = \frac{1}{4}(x^8 - 2 + x^{-8})$

In Exercises 9–14, find a definite integral that represents the arc length of the curve over the indicated interval. (Do not evaluate the integral.)

9. $y = x^2 + x - 2$ $[-2, 1]$

11. $y = 4 - x^2$ $[0, 2]$

13. $x = \frac{1}{y^2}$ $[1, 2]$

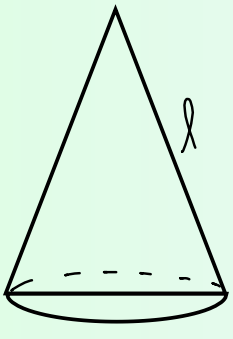
6.4 Arc length

Area of a surface of revolution

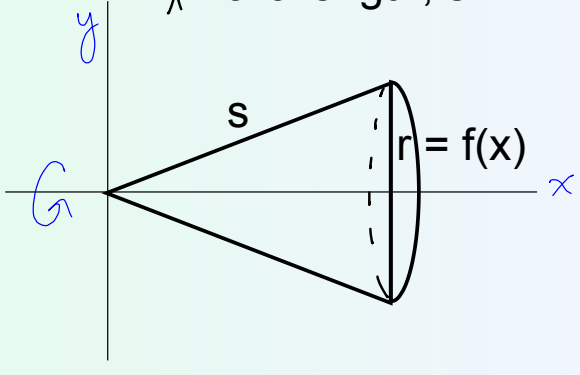
From Geometry

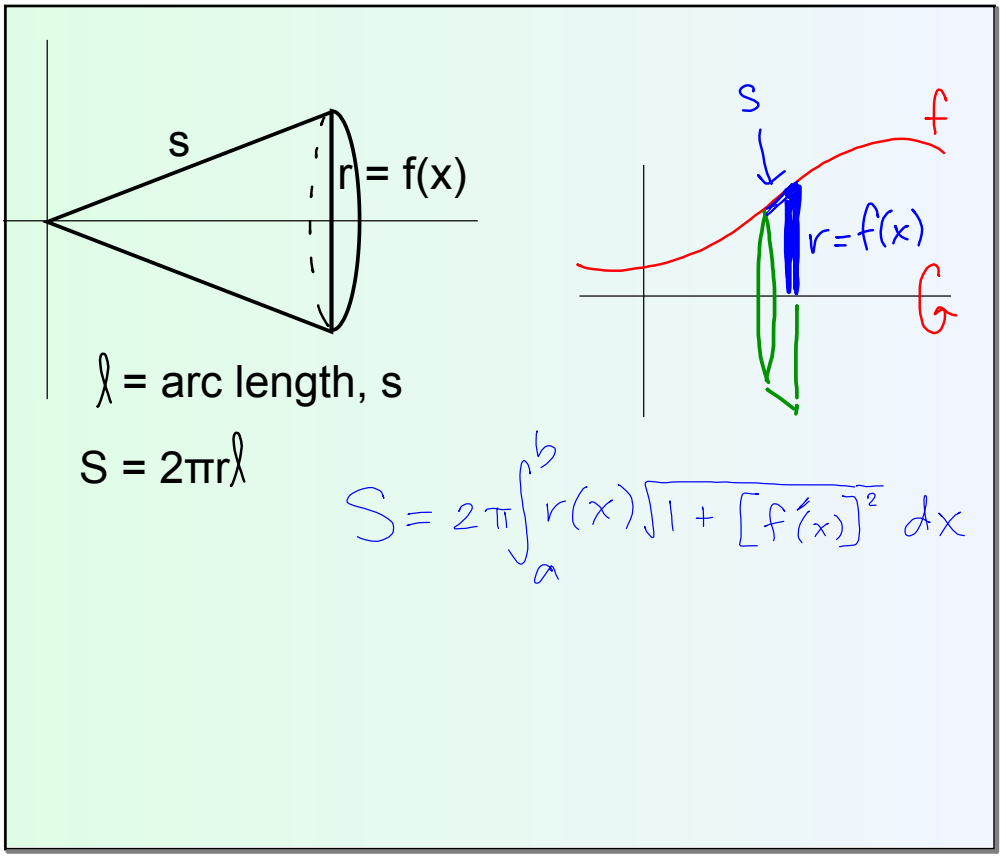
Surface area of a cone: $S = 2\pi r\ell$

ℓ = slant height



ℓ = arc length, s





DEFINITION OF SURFACE OF REVOLUTION

If the graph of a continuous function is revolved about a line, the resulting surface is called a **surface of revolution.**

DEFINITION OF THE AREA OF A SURFACE OF REVOLUTION

If $y = f(x)$ has a continuous derivative on the interval $[a, b]$, then the area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

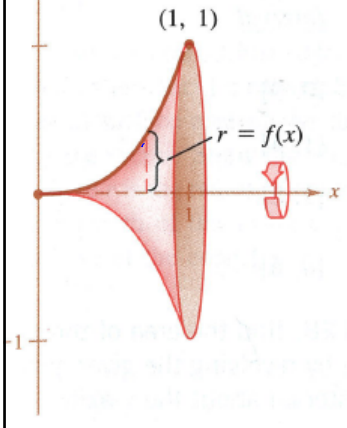
$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx$$

where $r(x)$ is the distance between the grapn of f and the axis of revolution.

Find the area of the surface formed by revolving f(x) about the x-axis on [0, 1].

$f(x) = x^3$

$f'(x) = 3x^2$



$$S = \frac{2\pi}{36}\int_0^1 36x^3\sqrt{1+9x^4}\,dx$$

$u = 1 + 9x^4$

$du = 36x^3\,dx$

$x = 0 \to u = 1$

$x = 1 \to u = 10$

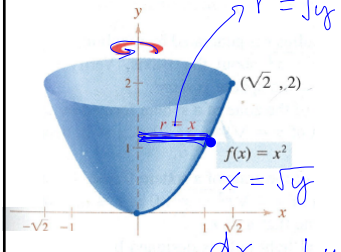
$$= \frac{\pi}{18}\int_1^{10} u^{1/2}\,du$$

$$= \frac{\pi}{18}\cdot\frac{2u^{3/2}}{3}\Big]_1^{10}$$

$$= \frac{\pi}{27}\left(10^{3/2} - 1\right)$$

$$\approx 3.563$$

Find the area of the surface formed by revolving f(x) about the y-axis on $[0, \sqrt{2}]$



$r = \sqrt{y}$

$x = \sqrt{y}$

$$\frac{dx}{dy} = \frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}}$$

$$S = 2\pi\int_0^2 \sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy$$

$$= 2\pi\int_0^2 \sqrt{y+\frac{1}{4}}\,dy$$

$u = y + \frac{1}{4}$ $\quad x = 0 \;\; u = \frac{1}{4}$

$du = dx$ $\quad x = 2 \;\; u = \frac{9}{4}$

$$= 2\pi\int_{1/4}^{9/4} u^{1/2}\,du$$

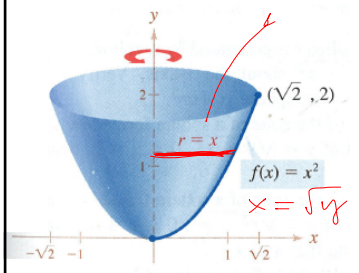
$$= 2\pi\cdot\frac{2u^{3/2}}{3}\Big]_{1/4}^{9/4}$$

$$= \frac{4\pi}{3}\left(\frac{9}{4}^{3/2} - \frac{1}{4}^{3/2}\right)$$

$$= \frac{4\pi}{3}\left(\frac{27}{8} - \frac{1}{8}\right)$$

$$= \boxed{\frac{13\pi}{3}}$$

Find the area of the surface formed by revolving f(x) about the y-axis on $[0, \sqrt{2}]$



$f'(x) = 2x$

$$S = \frac{2\pi}{8}\int_0^{\sqrt{2}} 8x\sqrt{1+4x^2}\,dx$$

$u = 1 + 4x^2$ $\quad x = 0,\ u = 1$

$du = 8x\,dx$ $\quad x = \sqrt{2},\ u = 9$

$$S = \frac{\pi}{4}\int_1^9 u^{1/2}\,du$$

$$= \frac{\pi}{4}\cdot\frac{2u^{3/2}}{3}\Big]_1^9$$

$$\frac{\pi}{6}\left(9^{3/2} - 1^{3/2}\right)$$

$$\frac{\pi}{6}(27 - 1)$$

$$\frac{13\pi}{3}$$

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21 - 27 odd

Individual Quiz: Thursday

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